dieser Faktor linear in die *minimale spezifische Schlitzlänge* ein, da der größere Wirkungsquerschnitt durch eine entsprechende Verminderung des Einlaßdruckes und damit des Gasdurchsatzes L kompensiert werden muß. Eine Veränderung der Düsenweite ist bei dieser Aufwandsgröße ohne Einfluß, da wegen der hierzu reziproken Änderung des optimalen Einlaßdruckes der Moldurchsatz der Düse unter optimalen Betriebsbedingungen unverändert bleibt. Beim *spezifischen Ansaugvolumen* läßt sich die negative Wirkung des größeren Wirkungsquerschnittes durch Verminderung der Düsenweite kompensieren. Da die Düsenweite bei der Geometrie B gegenüber der Düsenweite bei den Argonversuchen etwa auf die Hälfte reduziert wurde, ist es verständlich, daß bei dieser Aufwandsgröße eine kleinere Diskrepanz zwischen dem Ergebnis der Abschätzung und dem des Experimentes gefunden wird. Bei der *spezifischen Kompressionsarbeit* ist kein Einfluß des gaskinetischen Wirkungsquerschnittes auf den Minimalwert zu erwarten, da die ideale isotherme Kompressionsarbeit nur vom Druck*verhältnis*, nicht aber vom Absolutdruck abhängt. Tatsächlich stimmen bei dieser Aufwandsgröße nach Tab. 3 die gemessenen mit den abgeschätzten Werten nahezu überein [18].

Der in der vorliegenden Arbeit für UF_6 experimentell bestimmte Minimalwert der spezifischen idealen isothermen Kompressionsarbeit von $9{,}5 \cdot 10^2$ kWh/gMol ist um den Faktor 1,8 größer als der entsprechende, für das Diffusionsverfahren abgeschätzte Wert [19] von $5{,}4 \cdot 10^2$ kWh/gMol. Dieser Unterschied dürfte sich jedoch durch eine bessere Anpassung der Abschälgeometrie noch ausgleichen lassen [20]. Der schwächste Punkt des Trenndüsenverfahrens ist das große spezifische Ansaugvolumen, das verhältnismäßig hohe Investierungen für die Kompressoren und Rohrleitungen bedingt [21]. Andererseits stellt die Vermeidung der beim Diffusionsverfahren benutzten porösen Membranen einen betriebstechnischen Vorteil dar, der an anderen Stellen zu Kosteneinsparungen führen dürfte.

Bei der Durchführung der Anreicherungsversuche und bei den massenspektrometrischen Messungen haben Herr Dr. W. Henkes und Herr Dipl.-Phys. O. Hagena geholfen. Herrn Dr. K. Bier danken wir für wertvolle Diskussionen.

[18] Aus dem Vergleich scheint hervorzugehen, daß der Unterschied im Adiabatenexponenten von Argon und UF_6 ohne größere Bedeutung für die Entmischung ist. Dies beruht möglicherweise auf einer unvollständigen Akkommodation der inneren Freiheitsgrade des UF_6 während der mit nur wenigen Zusammenstößen verlaufenden Expansion.

[19] E. W. Becker, Chemie-Ing. Technik **29**, 365 [1957].

[20] Da bei der Vergrößerung des Abstandes zwischen Düse und Abschäler (Übergang von der Geometrie B zur Geometrie C) nach Tab. 2 alle Aufwandsgrößen ansteigen, kann erwartet werden, daß ein etwas kleinerer Abstand günstiger wäre. Dafür sprechen auch die früheren Versuche mit den Argonisotopen, bei denen die niedrigsten Werte der spezifischen Aufwandsgrößen bei einem Verhältnis Düsenweite zu Abstand Düse—Abschäler $\approx 1:1$ erreicht wurden.

[21] Das spezifische Ansaugvolumen kann theoretisch durch weitere Verminderung der kritischen Abmessungen des Trenndüsensystems herabgesetzt werden. In der Praxis ist jedoch eine untere Grenze durch mechanische Probleme gegeben.

The Potential Distribution in a Toroidal Condenser

By A. J. H. Boerboom

Laboratorium voor Massaspectrografie, Amsterdam, The Netherlands

(Z. Naturforschg. **15 a**, 347—350 [1960]; eingegangen am 13. Februar 1960)

The electrostatic potential is calculated in a toroidal condenser consisting of two rotational symmetric electrodes having the median plane as a plane of symmetry. The result is expressed as a series expansion in the coordinates around the main circle and in the mutual distance of the electrodes in the median plane.

Recently toroidal condensers have been described to be used for energy selecting devices in mass spectrographs [1]. In addition to the radial focusing they exert an axial focusing, in this way increasing the transmission. The problem of calculating the potential field in these condensers has been solved in

[1] H. Ewald and H. Liebl, Z. Naturforschg. **10 a**, 872 [1955]; **12 a**, 28 [1957]; **14 a**, 588 [1959]. H. Liebl and H. Ewald, Z. Naturforschg. **12 a**, 538, 541 [1957]; **14 a**, 199, 588, 842 [1959]. — H. Ewald, Z. Naturforschg. **14 a**, 198, 680 [1959]. — H. Ewald, H. Liebl and G. Sauermann, Z. Naturforschg. **14 a**, 129 [1959]. — G. Sauermann and H. Ewald, Z. Naturforschg. **14 a**, 137 [1959]. — H. Ewald and G. Sauermann, Z. Naturforschg. **11 a**, 173 [1956]. — H. Liebl, Z. Naturforschg. **13 a**, 490 [1958]. — H. Wachsmuth, H. Liebl and H. Ewald, Z. Naturforschg. **14 a**, 844 [1959]. — H. A. Tasman, Thesis, Leyden (in preparation).

principle by ALBRECHT [2]. By conformal mapping of the meridional cross section he reduces the problem to the solution of a number of linear equations. The method is applied on a toroidal condenser with circular intersections with a meridian plane and the coefficients of the series expansion of the potential are found up to the third order. Finding higher order terms seems tedious and the method is impracticable in the case of non circular cross sections.

The method of calculation, presented in this paper is similar to the method described to calculate the magnetic field between conical pole faces [3].

1. The electrostatic potential

The coordinate system is shown in Fig. 1, a meridional section is represented in Fig. 2. The dimensionless coordinates are (with r_e = radius of the main circle; the z-axis coincides with the axis of rotational symmetry)

$$u = (r - r_e)/r_e; \qquad v = z/r_e; \qquad w = \psi .$$

The distance of the electrodes in the median plane equals $2\,d$; the origin is taken at the main circle $(u = v = 0)$.

The shape of the potential field is supposed to be independent of w; the electrodes are supposed to be symmetrical with respect to the median plane.

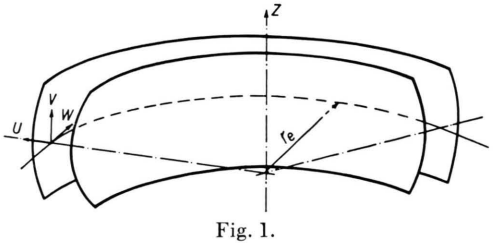

Fig. 1.

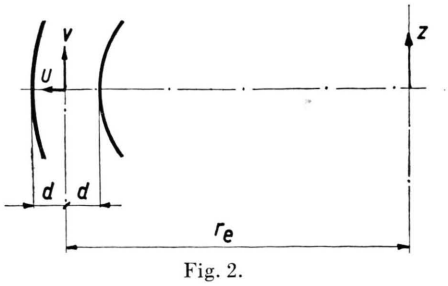

Fig. 2.

[2] R. ALBRECHT, Z. Naturforschg. **11 a**, 156 [1956].
[3] A. J. H. BOERBOOM, H. A. TASMAN and H. WACHSMUTH, Z. Naturforschg. **14 a**, 816 [1959].

The absolute value of the field strength $\partial\varphi/\partial r$ at $u = v = 0$ is designated by E_0, so the potential field may be expanded in a power series

$$\varphi_e/E_0\, r_e = \sum_{k,l=0}^{\infty} a_{k,l}\, u^k\, v^l , \tag{1}$$

where $a_{k,l} = 0$ for l = odd because of the symmetry and $a_{10} = 1$. In our coordinates the LAPLACE operator reads:

$$\nabla^2 = \frac{1}{r_e^2}\left\{\frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2} + \frac{\partial^2}{\partial w^2} + \frac{1}{1+u}\,\frac{\partial}{\partial u}\right\}. \tag{2}$$

Substitution of (1) in the Laplacian equation

$$\nabla^2\, \varphi_e/E_0\, r_e = 0$$

and equating the coefficient of the $u^{k+1}\, v^l$-term to zero provides the recurrence relation

$$(l+2)\,(l+1)\,(a_{k,l+2} + a_{k+1,l+2}) = -\,(k+3)\,(k+2)\,a_{k+3,l} - (k+2)^2\, a_{k+2,l}\ . \tag{3}$$

Choosing as independent parameters the coefficients $a_{0,l}$ and $a_{1,l}$, all other coefficients are determined through (3), together with the condition $a_{k,l} = 0$ for $k < 0$.

2. Toroidal electrodes

The electrode shapes in Fig. 2 can generally be represented by

$$u = d(1 + p_2\, v^2 + p_4\, v^4 + p_6\, v^6 + \ldots) \tag{4}$$

resp. $$u = -\,d(1 + q_2\, v^2 + q_4\, v^4 + q_6\, v^6 + \ldots) \tag{5}$$

(because of the supposed symmetry the odd terms in v are omitted). Some specializations of (4) or (5) are:

a) parabolic intersections, (4) resp. (5) reduce to:

$$u = d + p\, v^2 \quad \text{resp.} \quad u = -\,d - q\, v^2; \tag{6}$$

b) circular intersections, the radius of the circle being r_c:

$$u = d + \tfrac{1}{2}\,(v\, r_e/r_c)^2 + \tfrac{1}{8}\,(v\, r_e/r_c)^4 + \tfrac{1}{16}\,(v\, r_e/r_c)^6 + \ldots\ .$$

c) hyperbolic or ellipsoidal intersections:
If the hyperbola is given by

$$(u-m)^2 - v^2/b^2 = (d-m)^2,$$

(4) reduces to

$$u = d + \tfrac{1}{2}\,\frac{v^2}{b^2(d-m)} - \tfrac{1}{8}\,\frac{v^4}{b^4(d-m)^3} + \ldots\ ,$$

If the ellips is given by

$$(u-m)^2 + v^2/b^2 = (d-m)^2,$$

(4) reduces to

$$u = d - \tfrac{1}{2}\frac{v^2}{b^2(d-m)} - \tfrac{1}{8}\frac{v^4}{b^4(d-m)^3} + \dots .$$

For the following calculations any combination of electrode surfaces (p_i and q_i-values) is admitted.

The surfaces defined by (4) and (5) are equipotential surfaces. On substituting either (4) or (5) into (1) the result should identically equal a constant (corresponding to the potential of the electrode) and the sums of all terms in v^2, v^4, ... respectively should be individually zero. This yields relations between the $a_{k,l}$, which together with the relations (3), determine the $a_{k,l}$ as functions of d, p_i and q_i.

We expand $a_{k,l}$ in a power series in d:

$$a_{k,l} = \sum_{m=0}^{\infty} a_{k,l,m}\, d^m . \tag{7}$$

Now the $a_{k,l,m}$ are functions of the p_i and q_i, independent of d, which functions are directly found from linear equations.

For $l \neq 0$ the terms with $v^l d^0$ yield

$$a_{0,l,0} = 0; \tag{8}$$

the terms with $v^l d^1$ provide the recurrence relations

$$a_{1,l,0} = -\tfrac{1}{2}a_{1,l-2,0}(p_2+q_2) - \tfrac{1}{2}a_{1,l-4,0}(p_4+q_4) - \dots - \tfrac{1}{2}a_{1,0,0}(p_l+q_l), \tag{9}$$

$$a_{0,l,1} = -\tfrac{1}{2}a_{1,l-2,0}(p_2-q_2) - \tfrac{1}{2}a_{1,l-4,0}(p_4-q_4) - \dots - \tfrac{1}{2}a_{1,0,0}(p_l-q_l).$$

From (3) we find, together with (8)

$$a_{2,l,0} = -\tfrac{1}{2}a_{1,l,0}.$$

Now the terms with $v^l d^2$ ($l \neq 0$) yield $a_{1,l,1}$ and $a_{0,l,2}$, etc. In the case of parabolic equipotential surfaces ($p_i = q_i = 0$ for $i > 2$) the recurrence relations remain simple, for the general case it seems more appropriate to calculate directly the terms up to the desired order.

In this way one finds easily (including terms of the sixth order in u, v or d):

$$a_{10} = 1, \tag{10}$$

$$a_{20} = -\tfrac{1}{2} + \tfrac{1}{2}(p_2-q_2)\,d - \tfrac{1}{4}(p_2+q_2)\,d^2 + \{\tfrac{1}{4}(p_2-q_2) - \tfrac{3}{4}(p_2^2-q_2^2) + 3(p_4-q_4)\}\,d^3 + \{-\tfrac{5}{24}(p_2+q_2) - \tfrac{7}{24}(p_2+q_2)^2 + \tfrac{1}{2}(p_2-q_2)^2 - \tfrac{3}{2}(p_4+q_4)\}\,d^4 + \dots,$$

$$a_{30} = \{\tfrac{1}{3} + \tfrac{1}{6}(p_2+q_2)\} - \tfrac{1}{3}(p_2-q_2)\,d + \{\tfrac{7}{36}(p_2+q_2) - \tfrac{1}{12}(p_2+q_2)^2 + \tfrac{1}{6}(p_2-q_2)^2 + \tfrac{1}{3}(p_4+q_4)\}\,d^2 + \{-\tfrac{7}{36}(p_2-q_2) + \tfrac{5}{18}(p_2^2-q_2^2) - \tfrac{5}{3}(p_4-q_4)\}\,d^3 + \dots,$$

$$a_{40} = \{-\tfrac{1}{4} - \tfrac{1}{12}(p_2+q_2)\} + \{\tfrac{5}{24}(p_2-q_2) + \tfrac{1}{4}(p_2^2-q_2^2) - \tfrac{1}{2}(p_4-q_4)\}\,d + \{-\tfrac{17}{144}(p_2+q_2) + \tfrac{1}{24}(p_2+q_2)^2 - \tfrac{5}{24}(p_2-q_2)^2 + \tfrac{1}{12}(p_4+q_4)\}\,d^2 + \dots,$$

$$a_{50} = \{\tfrac{1}{5} + \tfrac{7}{120}(p_2+q_2) + \tfrac{1}{20}(p_2+q_2)^2 - \tfrac{1}{10}(p_4+q_4)\} + \{-\tfrac{19}{120}(p_2-q_2) - \tfrac{3}{20}(p_2^2-q_2^2) + \tfrac{3}{10}(p_4-q_4)\}\,d + \dots,$$

$$a_{60} = \{-\tfrac{1}{6} - \tfrac{11}{240}(p_2+q_2) - \tfrac{1}{40}(p_2+q_2)^2 + \tfrac{1}{20}(p_4+q_4)\} + \dots,$$

$$a_{02} = -\tfrac{1}{2}(p_2-q_2)\,d + \tfrac{1}{4}(p_2+q_2)\,d^2 + \{-\tfrac{1}{4}(p_2-q_2) + \tfrac{3}{4}(p_2^2-q_2^2) - 3(p_4-q_4)\}\,d^3 + \{\tfrac{5}{24}(p_2+q_2) + \tfrac{7}{24}(p_2+q_2)^2 - \tfrac{1}{2}(p_2-q_2)^2 + \tfrac{3}{2}(p_4+q_4)\}\,d^4 + \dots,$$

$$a_{12} = -\tfrac{1}{2}(p_2+q_2) + \tfrac{1}{2}(p_2-q_2)\,d + \{-\tfrac{1}{3}(p_2+q_2) + \tfrac{1}{4}(p_2+q_2)^2 - \tfrac{1}{2}(p_2-q_2)^2 - (p_4+q_4)\}\,d^2 + \{\tfrac{1}{3}(p_2-q_2) - \tfrac{1}{12}(p_2^2-q_2^2) + 2(p_4-q_4)\}\,d^3 + \dots,$$

$$a_{22} = \tfrac{1}{4}(p_2+q_2) + \{-\tfrac{1}{4}(p_2-q_2) - \tfrac{3}{2}(p_2^2-q_2^2) + 3(p_4-q_4)\}\,d + \{\tfrac{1}{6}(p_2+q_2) - \tfrac{1}{8}(p_2+q_2)^2 + (p_2-q_2)^2 - (p_4+q_4)\}\,d^2 + \dots,$$

$$a_{32} = \{-\tfrac{1}{6}(p_2+q_2) - \tfrac{1}{2}(p_2+q_2)^2 + (p_4+q_4)\} + \{\tfrac{1}{6}(p_2-q_2) + (p_2^2-q_2^2) - 2(p_4-q_4)\}\,d + \dots,$$

$$a_{42} = \{\tfrac{1}{8}(p_2+q_2) + \tfrac{1}{4}(p_2+q_2)^2 - \tfrac{1}{2}(p_4+q_4)\} + \dots,$$

$$a_{04} = \{\tfrac{1}{4}(p_2^2-q_2^2) - \tfrac{1}{2}(p_4-q_4)\}\,d + \{-\tfrac{1}{8}(p_2-q_2)^2 + \tfrac{1}{4}(p_4+q_4)\}\,d^2 + \dots,$$

$$a_{14} = \{\tfrac{1}{4}(p_2+q_2)^2 - \tfrac{1}{2}(p_4+q_4)\} + \{-\tfrac{1}{4}(p_2^2-q_2^2) + \tfrac{1}{2}(p_4-q_4)\}\,d + \dots,$$

$$a_{24} = \{-\tfrac{1}{8}(p_2+q_2)^2 + \tfrac{1}{4}(p_4+q_4)\} + \dots,$$

$$a_{06} = 0 + \dots .$$

The sixth order terms in (4) and (5) only enter in (10) into the terms of seventh and higher order.

3. Discussion

In the case of cylindrical electrodes ($p_i = q_i = 0$) the final results of **2** transform into the subsequent terms of the well known series expansion of the logarithmic potential

$$V(u, v) = V(0, 0) + u - \tfrac{1}{2} u^2 + \tfrac{1}{3} u^3 - \tfrac{1}{4} u^4 + - \dots .$$

Also in the case of the spherical condenser we get the initial terms of the corresponding potential.

The coefficients $a_{k,l}$ with $l \geqq 2$ contain no numerical constants and are either symmetrical (even terms in d) or antisymmetrical (odd terms in d) in p and q.

The potential of the electrodes is found from the v^0-terms in (1) by substituting $u = \pm d$:

$$V_{1,2}/E_0 r_e = a_{00} \pm a_{10} d + a_{20} d^2 \pm a_{30} d^3 + \dots .$$

The shape of electrodes, necessary to produce a desired type of field can be found by substituting (10) and putting $\varphi_e = \text{constant}$.

The parameters c and R_e' used in ion optical calculations [1] are connected with our parameters p and q through

$$c = r_e/R_e = -2\, a_{02}/a_{10} ,$$

$$R_e' = (a_{02}\, a_{20} - \tfrac{1}{2}\, a_{10}\, a_{12})/a_{02}^2 .$$

In exactly the same way the potential distribution can be found in rotational symmetric electrode systems, without a median plane of symmetry. In this case one has to take into account in (1) the odd terms in v. These terms will give results corresponding to the series expansions in [3], where the same series (1) is considered with $l = \text{odd}$.

The author is indebted to Prof. Dr. J. KISTEMAKER for his stimulating interest in this work, and to Mr. H. A. TASMAN for valuable discussions.

This work is part of the program of research of the Stichting voor Fundamenteel Onderzoek der Materie and was made possible by financial support of the Nederlandse Organisatie voor Zuiver Wetenschappelijk Onderzoek.

A modified Ion Slit Lens for virtual Variation of Slit Widths

By A. J. H. BOERBOOM

F. O. M.-Laboratorium voor Massaspectrografie, Amsterdam, Holland

(Z. Naturforschg. **15 a**, 350—355 [1960]; eingegangen am 13. Februar 1960)

It is shown that a slit lens system, already known in literature and used in mass spectrometer collector systems to vary the resolving power, can be greatly improved by introducing a potential on one of the electrodes, which previously was at zero potential. The focusing at the collector is unaltered within optical aberrations of the third order and the range of adjustment is increased as compared with the original system. Experimentally it was found that the virtual collector slit width could be adjusted from 1 mm down to 0.15 mm, maintaining a fair peak shape.

In 1953 CRAIG [1] published a slit lens system for use in mass spectrometer collector systems. It consists of four slits, as shown in Fig. 1. The first, third and fourth electrodes are at zero potential. The potential of the second electrode is varied. The first three electrodes act as an ion lens, whose strength is controlled in this way. Indeed CRAIG succeeded in getting an adjustable resolving power, corresponding to virtual collector slit widths ranging from 1.00 mm down to 0.25 mm.

At the higher voltages on the second electrode however, the peak shape is badly impaired. This effect is caused by the change in the location of the image of the first slit, when through the variation of the potential at the second electrode the strength

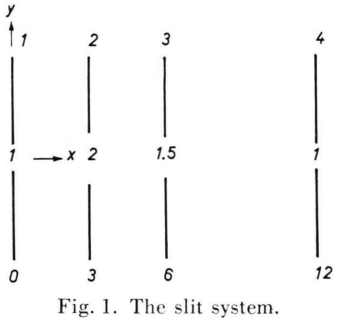

Fig. 1. The slit system.

of the lens is varied. For a proper action of the slit system this image should coincide with the fourth electrode. In this case the fourth slit will act as an

[1] R. D. CRAIG, Use of a retarding slit in mass spectrometer collector systems, Reports on the conference on applied mass spectrometry, London 1953, p. 230; Metro Vick Research Report No. 4987.